Statistical Inference For Rossby Waves: A Time Series Approach to Persistent Homology

Richard J Ross[1], Nicole Lazar[1], Lynne Seymour[1], Thomas L Mote[1]
[1]University of Georgia

Introduction

Topological data analysis (TDA) focuses on understanding the “shape” of data[1] , which provides a different perspective on previously unusable datasets.

Persistent Homology methods summarize homological features by calculating lifetimes (relative measure of persistence) for each feature

We apply Persistent Homology with the goal of not only identifying features, but also understanding how those features persist across large distances (Spatial Persistence), seasons, and years (Temporal Persistence).

Once the homology is summarized for a given point in time (i.e. a single day), time series describing how the homology changes can improve feature detection.

Analyzing these time series requires that one take into account the long range dependencies in the data while also detecting shifts which occur.

Rossby Waves

Rossby Waves are also called planetary waves. These waves are the large-scale north and south perturbations in the jet stream.

The number of waves naturally changes seasonally; Winter months exhibit fewer waves while summer months feature a higher number of waves.

When five or more Rossby waves are present in winter months, we often see weather extremes, like drought conditions (or flooding) in California recently.[3]

One difficulty in using TDA to identify waves lies in the “ridgeline” of the jet stream: We adapt Morse filtrations to accommodate this feature of isobaric data by performing sequential filtrations at each of many different latitude values.

The resulting summaries can be visualized and described by time series of the relative frequency of a given number of feature counts. These series hold information about the long-term behavior of the number of Rossby waves seen in the atmosphere.

Methods - Persistence

Since Rossby Waves tend to reside in the mid-latitudes, we perform sequential Morse filtrations on each band of latitude from 25°N – 65°N, giving us many filtrations for each date since 1950

Wave patterns are subsequently characterized by number of features present across latitudes. Lifetimes of specific wavenumber patterns are observed in “seahorse plots” (See Figure 1)

When a wavenumber pattern traverses many latitudes with long lifetimes, there is a higher degree of confidence that the pattern truly contains the indicated number of waves and is not thrown off by noisy data.

Region sizes for five wave formations are compiled day after day, creating a time series for each year since 1950 which indicates the relative confidence that five-wave formations are present. Figure 2 shows these series for two relevant time periods

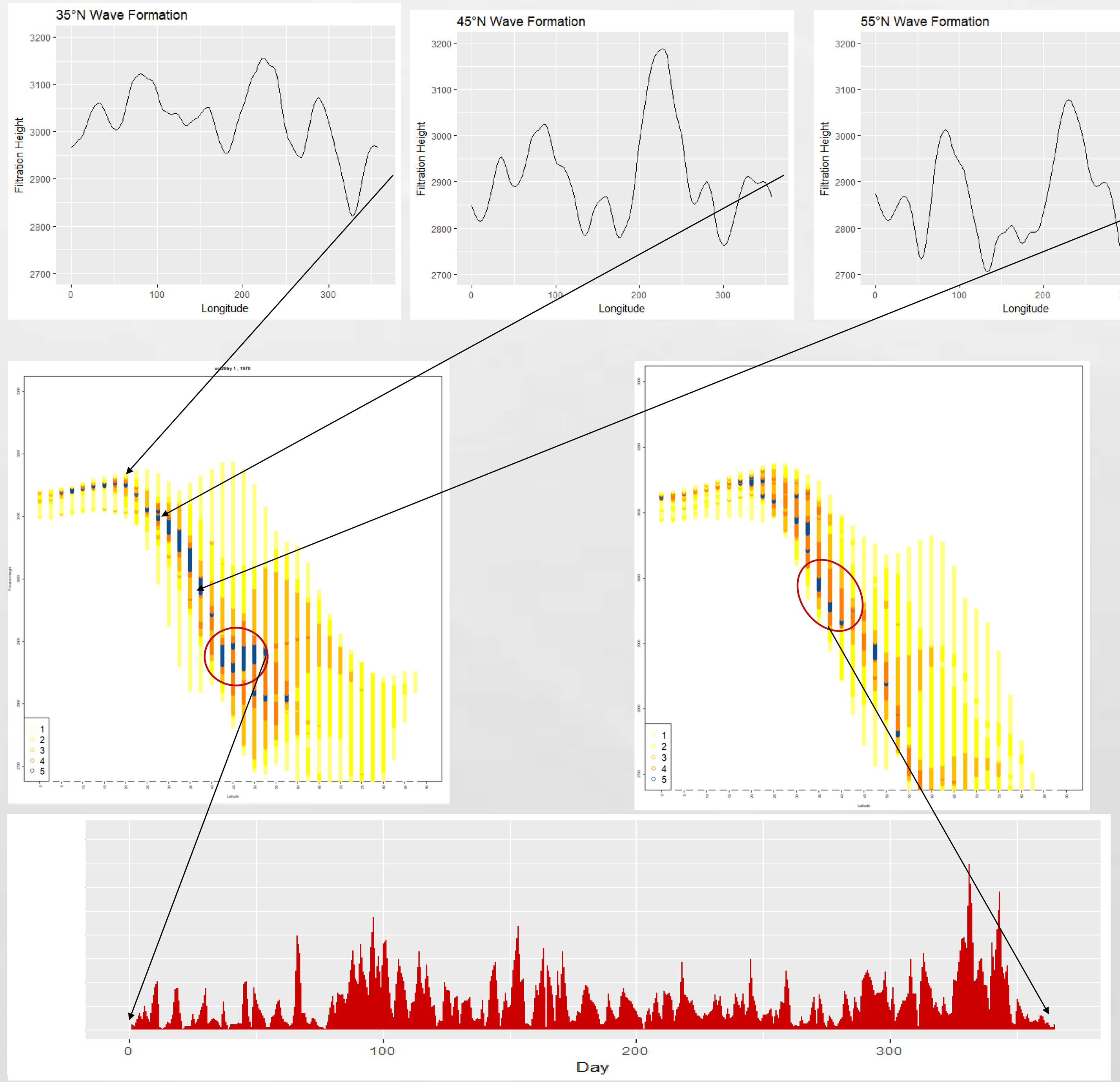

Figure 1. Workflow for Sequential Filtration. Individual waveforms are analyzed to form seahorse plot summaries, which form a time series of relative evidence for a wavenumber-5 formation

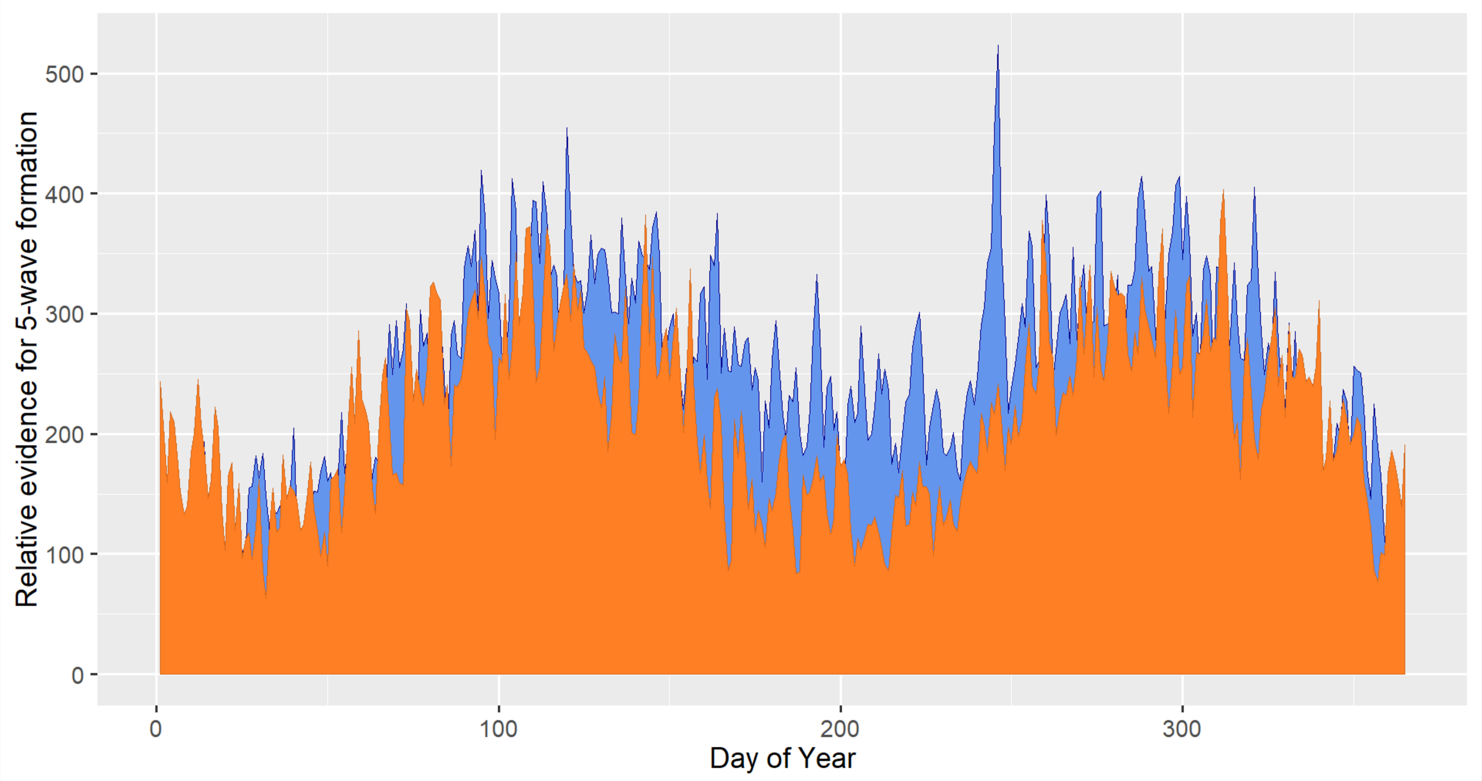

Figure 2. Comparison of average relative evidence for five-wave formations in the 20th midcentury (Orange) and early 21st century (Blue). Evidence is higher in more recent years, as suggested by theory regarding temperature gradients and wave numbers.

Discussion

Performing comparisons between these time series requires some pre-processing to ensure stationarity, but the results are very clear; There is increased evidence for elevated evidence of 5-wave formations in every season in recent years

Analysis performed using moving block bootstrapping. The bootstrap samples can be generated using fixed size blocks or random geometric block lengths, but the results are the same. For all seasons, present data provides significant evidence for increased evidence of five-wave formations ($P<0.01$ for all four seasons).

Conclusions

Sequential Morse filtration serves to help condense atmospheric data to time series which help identify the presence of global climate features.

Time series analysis of seahorse plot summaries provides significant evidence that Rossby waves have increased frequency from 1950 to 2015.

Spatial summaries are currently computed based on the size of the largest region in the seahorse plot where a given number of features are detected. Additional spatial summaries are under investigation to ensure robust results.

Persistent homology may serve as a reasonable pre-processing method for quantifying planetary features on the surface of other planetary bodies such as the Sun.

Contact

Richard J Ross
University of Georgia
Richard.ross25@uga.edu

References

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