



# Design and Analysis of Unreplicated Mixed Two-Level and Four-Level Split-Plot Type Experiments

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## INTRODUCTION

- Two-level factorial designs are widely used in practice.
- However, some applications may require factors with more than two-levels.
- Here, we focus on mixed two- and four-level designs.
- Construction of such completely randomized designs are found in the literature: for example, Wu and Zhang (1993); Ankenman (1999); Joseph, Mingyao and Wu (2009).
- We show a split-split-split-plot design with two- and four-level factors.
  - Criterion: minimum number of setups of harder-to-change factors
  - Extension of Ho, Vivacqua and Pinho (2012)

## EXAMPLE - BAJA COMPETITION EXPERIMENT

- The Society of Automotive Engineers (SAE) promotes the development of college students through car competitions all over the world.
- The objective of the experiment is to maximize the performance of the vehicle on two tests (acceleration and velocity) carried out on a paved street with an asphalt layer.

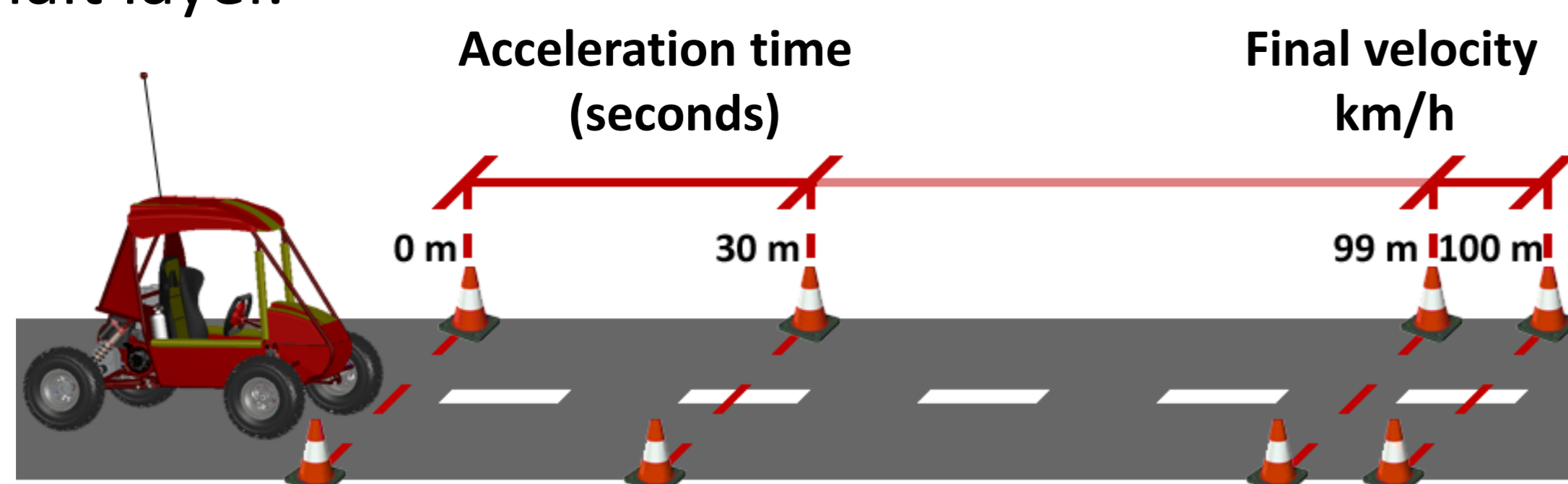


Figure 1. Illustration of the acceleration and velocity tests.

Table 1. Factors for the Baja competition experiment

Factor Label	Description of the Factor	# of Levels
A	Type of gear	2
B	Driven clutch springs	4
C	Pre-compression of the driven clutch spring	4
D	Drive clutch masses	4
E	Geometry of the ramp	2
F	Engine idle speed	4
G	Position of the gear ratio	2

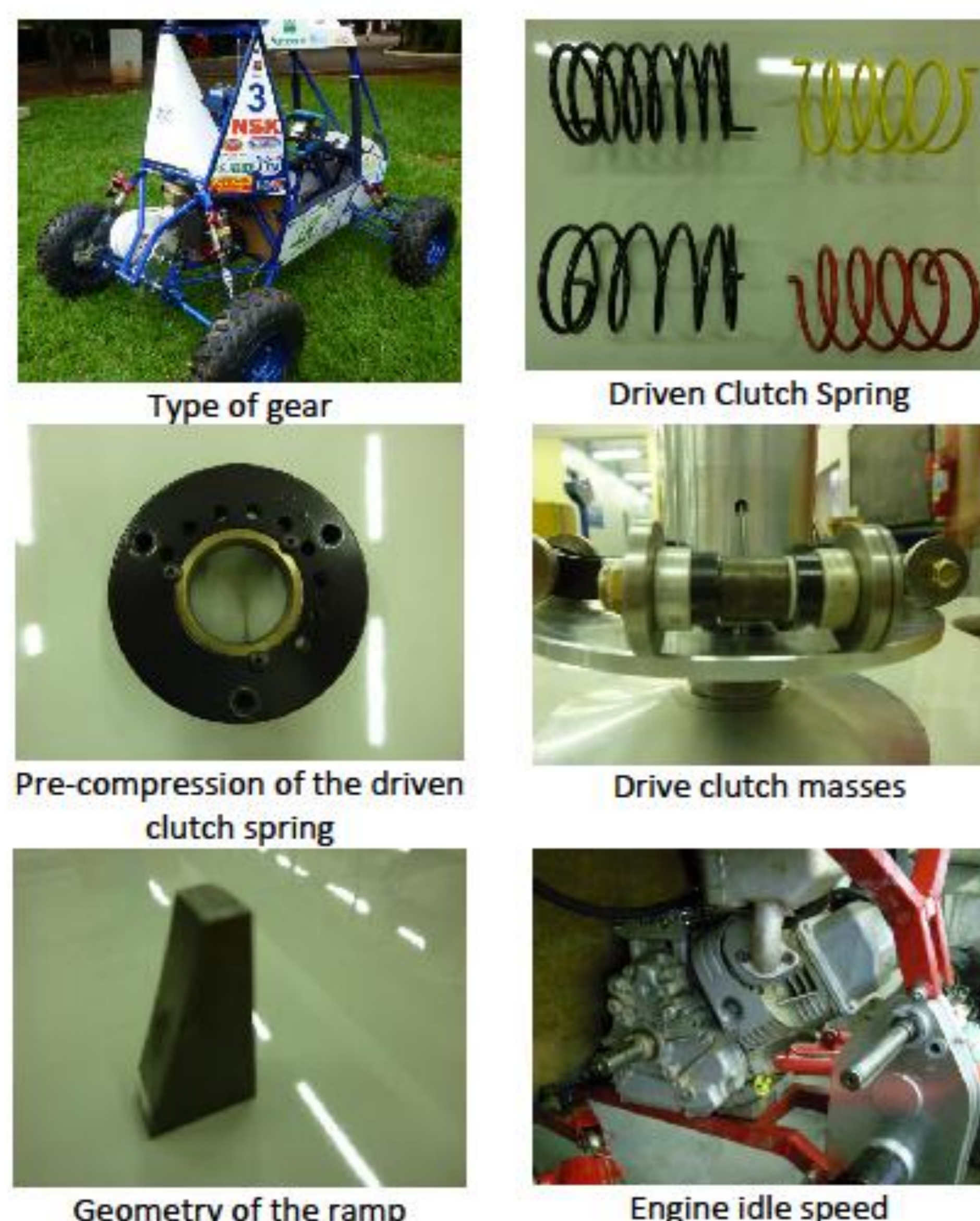


Figure 2. Auto parts used in the Baja experiment.

## SELECTING A DESIGN FOR THE BAJA COMPETITION EXPERIMENT

Table 2. Factors according to their degrees of difficulty in changing their levels

+ difficult	→		+ easy
Group 1	Group 2	Group 3	Group 4
Type of Gear (A)	Driven clutch springs (B)	Drive clutch masses (D)	Engine Idle Speed (F)
	Pre-compression of driven clutch springs (C)	Geometry of the ramp (E)	Position of the gear ratio (G)

- We use the catalogs proposed by Ho, Vivacqua and Pinho (2012) for two-level split-plot designs and adapt it to the mixed case.
- Two-level reference design:  $2^{(1-0)} \times 2^{(4-2)} \times 2^{(3-2)} \times 2^{(3-2)}$
- Number of setups at each stratum: 2, 8, 16, 32

## ANALYSIS

- The analysis should be conducted on a stratum-by-stratum basis.
- For unreplicated designs, one alternative is to use normal probability plots, keeping in mind that only effects with the same variance should be plotted on the same normal or half-normal plot.

## CONCLUSION

- In physical prototype testing experiments with hard-to-change factors, split-plot type designs represent a cost-effective method for the generation of information to guide the decision-making process.
- Some designs from catalogs of two-level split-plot type designs may be used for constructing mixed two- and four-level split-plot type designs.

## REFERENCES

- Ankenman, B. E. (1999) Design of experiments with two- and four level factors, *Journal of Quality Technology*, v. 31 (4), 363-375.
- Ho, L. L., Vivacqua, C. A., Pinho, A. L. S. (2012) Catalogs of split-plot type designs for physical prototype testing, *Design and Analysis of Experiments Conference*, Athens, GA, USA.
- Joseph, V. R., Mingyao, A., Wu, C. F. J. (2009) Bayesian-inspired minimum aberration two- and four-level designs, *Biometrika*, v. 96 (1), 95-106.
- Wu, C. F. J., Zhang, R. (1993) Minimum aberration designs with two-level and four-level factors, v. 80 (1), 203-209.

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