The Problem and Objective

The Problem:
- There are hundreds of definitions for heatwaves based on temperature and humidity measures
- BUT... almost all are at best ad hoc (i.e., not based on probabilistic framework)

Objective: Build a definition of Heatwave based on a probabilistic inferential framework.

What are Heatwaves?

Some examples of existing definitions of heatwaves (out of >90 available in literature):

- Frich et al. 2002: At least 5 consecutive days, the maximum temperature exceeds the normal temperature by 5°C (9°F) based on a period of 1961-1990.
- Heat Index (HI): A nonlinear function of Relative Humidity and Temperature.
  - Caution: HI is 80-90°F
  - Extreme Caution: HI is 90-103°F
  - Danger: HI is 103-125°F
  - Extreme Danger: HI over 125°F
- Xu et al. (2018) evaluated 29 different definitions of Heatwave
- Vaidyanathan et al. (2016) explored 92 different definitions

Probabilistic Framework

- $X_1, X_2, \ldots$: A strictly stationary series
- $M$: Threshold (location specific)
- $T_1, T_2, \ldots$: Times of the up-crossings
- $T_2, T_4, \ldots$: Times of the down-crossings
- Duration: $D_k = T_{2k} - T_{2k-1}$, $k = 1, 2, \ldots$
- Intensity: $X_{T_{2k} - 1} + \ldots + X_{T_{2k} - 1} - M(T_{2k} - T_{2k-1})$

Hierarchical model

Let $B_k = \mathbb{I}(X_t > M)$. Under a set of regularity conditions [1], the distribution of $D_k$ can be approximated by

$$D_k \approx \text{Geo}(e^{-pB_k(1)})$$

Distribution of Duration (Exact)

Assume that $X_1, X_2, \ldots$ follows $AR_p(\theta)$, $p$-th order stationary AR process with parameter $\theta$. Then

$$P(D_k = l) = \frac{\pi_2(\theta)}{A(\theta) \times B(\theta)^{l-p}} \quad l < p$$

for $l = 1, 2, \ldots$

USCRN Data Analysis

- USCRN Data: 18 years (2000-2017) of meteorology data (daily average temperature) across 120 weather stations in USA and around.

Case Studies

- Atlanta Data: 22 years (1991-2012) of meteorology data (daily maximum temperature)

Atlanta Data Analysis

- Expected length = 23.48 - 31.96 $q + 10.87 q^2$
  (where $q = P(X_t \leq M) \in (0.70, 0.95)$)

Figure 3: Quadratic fit to the expected lengths with Adjusted $R^2 = 0.96$

References


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